The internally assessed component in these courses is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection. A student should develop his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow all students to develop an area of interest for them, without a time constraint as in an examination, and will allow all to experience a feeling of success.

In addition to testing the objectives of the courses, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the courses, in particular aims 6–9 (applications, technology, moral, social and ethical implications, and the international dimension). It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile.

Skills and strategies required by students

The exploration is a significant part of the course. It is useful to think of it as a developing piece of work, which requires particular skills and strategies. As a general rule, it is unrealistic to expect all students to have these specific skills and to follow particular strategies before commencing the course.

Many of the skills and strategies identified below can be integrated into the course of study by applying them to a variety of different situations both inside and outside the classroom. In this way, students can practise certain skills and learn to follow appropriate strategies in a more structured environment before moving on to working independently on their explorations.

Choosing a topic

- Identifying an appropriate topic
- Developing a topic
- Devising a focus that is well defined and appropriate
- Ensuring that the topic lends itself to a concise exploration

Communication

- Expressing ideas clearly
- Identifying a clear aim for the exploration
- Focusing on the aim and avoiding irrelevance
- Structuring ideas in a logical manner
- Including graphs, tables and diagrams at appropriate places
- Editing the exploration so that it is easy to follow
- Citing references where appropriate

Mathematical presentation

- Using appropriate mathematical language and representation
- Defining key terms, where required
- Selecting appropriate mathematical tools (including information and communication technology)
- Expressing results to an appropriate degree of accuracy
Personal engagement

- Working independently
- Asking questions, making conjectures and investigating mathematical ideas
- Reading about mathematics and researching areas of interest
- Looking for and creating mathematical models for real-world situations
- Considering historical and global perspectives
- Exploring unfamiliar mathematics

Reflection

- Discussing the implications of results
- Considering the significance of the exploration
- Looking at possible limitations and/or extensions
- Making links to different fields and/or areas of mathematics

Use of mathematics

- Demonstrating knowledge and understanding
- Applying mathematics in different contexts
- Applying problem-solving techniques
- Recognizing and explaining patterns, where appropriate
- Generalizing and justifying conclusions

Stimuli

Students sometimes find it difficult to know where to start with a task as open-ended as this. While it is hoped that students will appreciate the richness of opportunities for mathematical exploration, it may sometimes be useful to provide a stimulus as a means of helping them to get started on their explorations.

Possible stimuli that could be given to the students include:

<table>
<thead>
<tr>
<th>sport</th>
<th>archaeology</th>
<th>computers</th>
<th>algorithms</th>
<th>algorithms</th>
</tr>
</thead>
<tbody>
<tr>
<td>cell phones</td>
<td>music</td>
<td>sine</td>
<td>musical harmony</td>
<td>musical harmony</td>
</tr>
<tr>
<td>motion</td>
<td>e</td>
<td>electricity</td>
<td>water</td>
<td>water</td>
</tr>
<tr>
<td>space</td>
<td>orbits</td>
<td>food</td>
<td>volcanoes</td>
<td>volcanoes</td>
</tr>
<tr>
<td>diet</td>
<td>Euler</td>
<td>games</td>
<td>symmetry</td>
<td>symmetry</td>
</tr>
<tr>
<td>architecture</td>
<td>codes</td>
<td>the internet</td>
<td>communication</td>
<td>communication</td>
</tr>
<tr>
<td>tiling</td>
<td>population</td>
<td>agriculture</td>
<td>viruses</td>
<td>viruses</td>
</tr>
<tr>
<td>health</td>
<td>dance</td>
<td>play</td>
<td>pi (π)</td>
<td>pi (π)</td>
</tr>
<tr>
<td>geography</td>
<td>biology</td>
<td>business</td>
<td>economics</td>
<td>economics</td>
</tr>
<tr>
<td>physics</td>
<td>chemistry</td>
<td>information technology in a global society</td>
<td>psychology</td>
<td>psychology</td>
</tr>
</tbody>
</table>
Frequently asked questions

What is the difference between a mathematical exploration and an extended essay in mathematics?

The criteria are completely different. It is intended that the exploration is to be a much less extensive piece of work than a mathematics extended essay. The intention is for students to “explore” an idea rather than have to do the formal research demanded in an extended essay.

How long should it be?

It is difficult to be prescriptive about mathematical writing. However, the Mathematics SL guide and the Mathematics HL guide state that 6–12 pages should be appropriate. A common failing of mathematical writing is excessive repetition, and this should be avoided, as such explorations will be penalized for lack of conciseness. However, it is recognized that some explorations will require the use of several diagrams, which may extend them beyond the page limit.

Does the exploration need a title?

It is good practice to have a title for all pieces of work. If the exploration is based on a stimulus, it is recommended that the title not just be the stimulus. Rather, the title should give a better indication of where the stimulus has taken the student. For example, rather than have the title “water”, the title could be “Water—predicting storm surges”.

Can students in the same school/class use the same title for the exploration?

Yes, but the explorations must be different, based on the avenues followed by each student. As noted above, the title should give an idea of what the exploration is about. Group work is not allowed.

Can students in the same school/class use the same stimulus?

Yes, this is permissible. However, the stimuli are intended to be broad themes around which a variety of foci could develop. It is therefore expected that, even if students use the same stimuli, the resulting explorations will be very different.

Can SL and HL students use the same stimulus?

Yes, there is no reason to restrict any stimulus to a particular level, although the assessment of criterion E will be different.

Do teachers have to use stimuli?

No, but it may sometimes be useful to provide a stimulus as a means of helping students to get started on their exploration.

Should the scope and sequence of the SL/HL course be influenced by the exploration?

Ideally, it should not be. It is intended that the exploration should be a natural opportunity to develop ideas that students have become familiar with as a part of the course. However, if it is felt that particular skills are likely to be needed in order for students to undertake the exploration successfully, then a teacher or school may wish to consider this when deciding on the teaching sequence.
How much help can a teacher give the student in finding a topic/focus for their exploration?

The role of the teacher here is to provide advice to the student on choosing the topic, and there is no set limit to the amount of help a teacher can give in this respect. However, if the student has little or no input into the decision about which focus to choose, then it is unlikely that he or she will be able to explore the ideas successfully in order to generate a good exploration.

How much help can the teacher give to the student with the mathematical content of the exploration?

If a student needs help with the revision of a particular topic because they are having some problems using this in their exploration, then it is permissible (indeed, this is good practice) for the teacher to give this help. However, this must be done in such a way that is not directly connected with the exploration.

What should the target audience be for a student when writing the exploration?

The exploration should be accessible to fellow students.

Can the students use mathematics other than that they have done in class?

Yes, but this must be clearly explained and referenced, and teacher comments should clarify this.

Can students use mathematics that is outside the syllabus?

Yes, as long as the mathematics used is relevant. However, this is not necessary to obtain full marks.

What is the difference between criterion A (communication) and criterion B (mathematical presentation)?

Communication is focusing on the overall organization and coherence of the exploration, whereas mathematical presentation focuses on the appropriateness of the mathematics. An exploration that is logically set out in terms of its overall structure could score well in criterion A despite using inappropriate mathematics. Conversely, an exploration that uses appropriate diagrams and technology to develop the ideas could score well in criterion B but poorly in criterion A because it lacked a clear aim or conclusion, for example.

Does the exploration have to be word processed or handwritten?

It can be in either form as long as it is clearly legible.

What is personal engagement?

The exploration is intended to be an opportunity for students to use mathematics to develop an area of interest to them rather than merely to solve a problem set by someone else. Criterion C (personal engagement) will be looking at how well the student is able to demonstrate that he or she has “made the exploration their own” and expressed ideas in an individual way.

What is the difference between precise and correct?

As outlined in criterion E (use of mathematics), “precise” mathematics requires absolute accuracy with appropriate use of notation. “Correct” mathematics may contain the occasional error as long as it does not seriously interfere with the flow of the work or give rise to conclusions or answers that are clearly wrong.
Maths IA – Maths Exploration Topics

The authors of the latest Pearson Mathematics SL and HL books – (which look really good) have come up with 200 ideas for students doing their maths explorations. These topics touch a really large cross-section of mathematics. A lot of these ideas would need the students to go away and research independently to find out about them – which in itself would be a nice activity. There’s also a lot of potential for integrating some of this content into maths lessons and ToK discussions.

It is essential that you read the following guidance from the IB prior to starting your IA maths exploration – this linked site gives the full list of assessment criteria you will be judged against and also gives 9 full examples of investigations students have done.

**Algebra and number theory**

1) Modular arithmetic
2) Goldbach’s conjecture: “Every even number greater than 2 can be expressed as the sum of two primes.” One of the great unsolved problems in mathematics.
3) Probabilistic number theory
4) Applications of complex numbers: The stunning graphics of Mandelbrot and Julia Sets are generated by complex numbers.
5) Diophantine equations: These are polynomials which have integer solutions. Fermat’s Last Theorem is one of the most famous such equations.
6) Continued fractions: These are fractions which continue to infinity. The great Indian mathematician Ramanujan discovered some amazing examples of these.
7) General solution of a cubic equation
8) Applications of logarithms
9) Polar equations
10) Patterns in Pascal’s triangle: There are a large number of patterns to discover – including the Fibonacci sequence.
11) Finding prime numbers: The search for prime numbers and the twin prime conjecture are some of the most important problems in mathematics. There is a $1 million prize for solving the Riemann Hypothesis and $250,000 available for anyone who discovers a new, really big prime number.
12) Random numbers
13) Pythagorean triples: A great introduction into number theory – investigating the solutions of Pythagoras’ Theorem which are integers (eg. 3,4,5 triangle).
14) Mersenne primes: These are primes that can be written as $2^n -1$.
15) Magic squares and cubes
16) Loci and complex numbers
17) Matrices and Cramer’s rule
18) Divisibility tests
19) Egyptian fractions: Egyptian fractions can only have a numerator of 1 – which leads to some interesting patterns. 2/3 could be written as 1/6 + 1/2. Can all fractions with a numerator of 2 be written as 2 Egyptian fractions?
20) Complex numbers and transformations
21) Euler’s identity: An equation that has been voted the most beautiful equation of all time, Euler’s identity links together 5 of the most important numbers in mathematics.
22) Chinese remainder theorem
23) **Fermat’s last theorem**: A problem that puzzled mathematicians for centuries – and one that has only recently been solved.
24) Natural logarithms of complex numbers
25) **Twin primes problem**: The question as to whether there are patterns in the primes has fascinated mathematicians for centuries. The twin prime conjecture states that there are infinitely many consecutive primes (eg. 5 and 7 are consecutive primes). There has been a recent breakthrough in this problem.
26) Hypercomplex numbers
27) Diophantine application: Cole numbers
28) **Odd perfect numbers**: Perfect numbers are the sum of their factors (apart from the last factor). ie 6 is a perfect number because 1 + 2 + 3 = 6.
29) Euclidean algorithm for GCF
30) **Palindrome numbers**: Palindrome numbers are the same backwards as forwards.
31) Algebraic congruences
32) Inequalities related to Fibonacci numbers
33) Combinatorics – art of counting
34) Boolean algebra
35) Graphical representation of roots of complex numbers
36) **Fermat’s little theorem**: If p is a prime number then a^p – a is a multiple of p.
37) Prime number sieves
38) **Recurrence expressions for phi** (golden ratio): Phi appears with remarkable consistency in nature and appears to shape our understanding of beauty and symmetry.
39) **The Riemann Hypothesis** – one of the greatest unsolved problems in mathematics – worth $1 million to anyone who solves it (not for the faint hearted!)

**Geometry**

![Image of geometric shape]

1) **Non-Euclidean geometries**: This allows us to “break” the rules of conventional geometry – for example, angles in a triangle no longer add up to 180 degrees.
2) Cavalieri’s principle
3) Packing 2D and 3D shapes
4) Ptolemy’s theorem
5) **Hexaflexagons**: These are origami style shapes that through folding reveal some amazing properties. Can you find all the hidden faces?
6) Heron’s formula
7) Geodesic domes
8) Proofs of Pythagorean theorem
9) **Minimal surfaces and soap bubbles**: Soap bubbles assume the minimum possible surface area to contain a given volume.
10) **Tesseract – a 4D cube**: How we can use maths to imagine higher dimensions.
11) Map projections
12) Tiling the plane – tessellations

Penrose tiles: Penrose tiles produce non-symmetrical but self-similar patterns.

13) Morley’s theorem
14) Cycloid curve
15) Symmetries of spider webs
16) Fractal tilings
17) Euler line of a triangle
18) Fermat point for polygons and polyhedra
19) Pick’s theorem and lattices
20) Properties of a regular pentagon
21) Conic sections
22) Nine-point circle
23) Geometry of the catenary curve
24) Regular polyhedra
25) Euler’s formula for polyhedra

26) Stacking cannon balls: An investigation into the patterns formed from stacking cannon balls in different ways.
27) Ceva’s theorem for triangles
28) Constructing a cone from a circle
29) Conic sections as loci of points
30) Consecutive integral triangles

31) Area of an ellipse
32) Mandelbrot set and fractal shapes: Explore the world of infinitely generated pictures and fractional dimensions.
33) Curves of constant width
34) Sierpinski triangle: a fractal design that continues forever.
35) Squaring the circle: This is a puzzle from ancient times – which was to find out whether a square could be created that had the same area as a given circle. It is now used as a saying to represent something impossible.

36) Polyominoes: These are shapes made from squares. The challenge is to see how many different shapes can be made with a given number of squares – and how can they fit together?
37) Reuleaux triangle
38) Architecture and trigonometry
39) Spherical geometry
40) Gyroid – a minimal surface
41) Geometric structure of the universe
42) Rigid and non-rigid geometric structures

43) Tangrams: Investigate how many different ways different size shapes can be fitted together.
44) Understanding the fourth dimension: How we can use mathematics to imagine (and test for) extra dimensions.

Calculus/analysis and functions
1) Mean value theorem
2) Torricelli’s trumpet (Gabriel’s horn): An amazing shape which has a finite volume but an infinite surface area.
3) Integrating to infinity
4) Applications of power series
5) Newton’s law of cooling
6) Fundamental theorem of calculus
7) Brachistochrone (minimum time) problem
8) Second order differential equations
9) L'Hôpital’s rule and evaluating limits
10) Hyperbolic functions
11) The harmonic series: Investigate the relationship between fractions and music, or investigate whether this series converges.
12) Torus – solid of revolution: A torus is a donut shape which introduces some interesting topological ideas.
13) Projectile motion: Studying the motion of projectiles like balls and rockets is an essential skill for physicists. A good use of your calculus skills.
14) Why e is base of natural logarithm function: A chance to investigate both natural logs and the amazing number e.

Statistics and modelling

1) Traffic flow: How maths can model traffic on the roads.
2) Logistic function and constrained growth
3) Benford’s Law - using statistics to catch criminals by making use of a surprising distribution.
4) Bad maths in court - how a misuse of statistics in the courtroom can lead to devastating miscarriages of justice.
5) The mathematics of cons – how con artists use pyramid schemes to get rich quick.
6) Impact Earth - what would happen if an asteroid or meteorite hit the Earth?
7) Black Swan events – how useful can mathematics predict small probability high impact events?
8) Modelling happiness – how understanding utility value can make you happier.
9) Does finger length predict mathematical ability? Investigate the surprising correlation between finger ratios and all sorts of abilities and traits.
10) Modelling epidemics/spread of a virus
11) Modelling the shape of a bird’s egg
12) Modelling change in record performances for a sport
13) Modelling radioactive decay
14) Modelling the carrying capacity of the earth
15) Regression to the mean
16) Modelling growth of computer power past few decades
17) Probability and probability distributions
18) The Monty Hall problem
19) Monte Carlo simulations
20) Random walks
21) Insurance and calculating risks
22) Poisson distribution and queues
23) Lotteries
24) Bayes’ theorem: How understanding probability is essential to our legal system.
25) Birthday paradox: The birthday paradox shows how intuitive ideas on probability can often be wrong. How many people need to be in a room for it to be at least 50% likely that two people will share the same birthday? Find out!
26) Normal distribution and natural phenomena
27) Medical tests and probability
28) Probability and expectation
1) **The prisoner’s dilemma**: The use of game theory in psychology and economics.
2) Sudoku
3) **Gambler’s fallacy**: A good chance to investigate misconceptions in probability and probabilities in gambling. Why does the house always win?
4) Poker and other card games
5) **Knight’s tour in chess**: This chess puzzle asks how many moves a knight must make to visit all squares on a chess board.
6) Billiards and snooker
7) Zero sum games

**Topology and networks**

1) Knots
2) Steiner problem
3) Chinese postman problem
4) Travelling salesman problem
5) **Königsberg bridge problem**: The use of networks to solve problems. This particular problem was solved by Euler.
6) **Handshake problem**: With n people in a room, how many handshakes are required so that everyone shakes hands with everyone else?
7) Möbius strip
8) Klein bottle
9) Logic and sets
10) **Codes and ciphers**: ISBN codes and credit card codes are just some examples of how codes are essential to modern life. Maths can be used to both make these codes and break them.
11) Set theory and different ‘size’ infinities
12) Mathematical induction (strong)
13) Proof by contradiction
14) **Zeno’s paradox of Achilles and the tortoise**: How can a running Achilles ever catch the tortoise if in the time taken to halve the distance, the tortoise has moved yet further away?
15) Four colour map theorem
16) Numerical analysis
17) Linear programming
18) Fixed-point iteration
19) Applications of iteration
20) Newton’s method
21) Estimating size of large crowds
22) Generating the number e
23) Descartes’ rule of signs
24) Methods for solving differential equations

**Physical, biological and social sciences**

Radiocarbon dating
Gravity, orbits and escape velocity
Mathematical methods in economics
Biostatistics
Genetics
Crystallography
Computing centres of mass
Elliptical orbits
Logarithmic scales – decibel, Richter, etc.
Fibonacci sequence and spirals in nature
Predicting an eclipse
Change in a person’s BMI over time
Concepts of equilibrium in economics
Mathematics of the ‘credit crunch’ Branching patterns of plants
Column buckling – Euler theory
Miscellaneous

Paper folding
Designing bridges
Mathematics of rotating gears
Mathematical card tricks
Curry’s paradox – ‘missing’ square
Bar codes
Applications of parabolas
Music – notes, pitches, scales...
Voting systems
Flatland by Edwin Abbott
Terminal velocity
Towers of Hanoi puzzle
Photography
Art of M.C. Escher
Harmonic mean
Sundials
Navigational systems
The abacus
Construction of calendars
Slide rules
Different number systems
Mathematics of juggling
Global positioning system (GPS)
Optical illusions
Origami
Napier’s bones
Celtic designs/knotwork
Design of product packaging
Mathematics of weaving