

WHAT TO EXPECT ON THE CALCULUS BC FREE RESPONSE QUESTIONS FOR MAY 7TH, 2014

1 Infinite Series (Power, Taylor, Maclaurin)

- Taylor: $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{iv}(a)}{4!}(x-a)^4 + \dots$
- When $a=0$, it is called a Maclaurin series.
- Memorize the ones you should have memorized!!!!
- Does the series converge or diverge? (Use Convergence Tests)
- Estimate using an n th degree Taylor Series
- Usually use the **Ratio Test** to find Interval of convergence
- Radius of convergence = $(1/2)(\text{Interval of convergence})$
- If alternating, then $|\text{error}| < \text{"next term"}$
- Lagrange "error bound" $[|\text{error}| < \text{"next term" but with conditions}]$

2 Area and Volume Problem

- Calculate Area and Calculate Volume
- Rotated around x-axis and Rotated around y-axis
- Rotated around line $x = k$ and Rotated around line $y = k$
- Calculate Volume of Cross Section

Special note: This will probably be one of the four NON-CALCULATOR problems, although it isn't always on the BC test. This is typically the easiest (and most predictable) of the Free Response questions. Make sure you know how to: Zoom Fit, Adjust Window, Store Values, use VARS in case it is a calculator problem (unlikely) Your goal is to get a NINE on this one!

3 Polar Equations

- Area of a sector = $\frac{1}{2}r^2\theta \Rightarrow$

$\text{Area} = \frac{1}{2} \int_a^b r^2 d\theta$
- $x = r \cos \theta$ AND $y = r \sin \theta$ AND $r = \pm \sqrt{x^2 + y^2}$ AND $\tan \theta = \frac{y}{x}$
- Use your calculator to solve for a and b [TRACE].
If non-calculator, set r equal to zero and solve to find a and b .

4 Analyzing a Graph of a Derivative

- Graph could be given as **f' (derivative)**
- Graph could be given as **velocity** [derivative of $x(t)$]
- Graph could be given as **2nd Fundamental Theorem of Calculus: {MOST LIKELY}**
 - For example, graph of $f(x)$ is given and $g(x) = \int_c^x f(t) dt$
 - Thus, $g'(x)$ would then equal $f(x)$ so graph is REALLY $g'(x)$

Special note: Look at Particle Problem and Detective's Hat Problem (Piecewise Graph)

5 Parametric Equations

- Arc length – know all 3 formulas

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

- Speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$
- Total Distance = $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \Rightarrow$ Notice that this is the same as parametric arc length.
(This is derived from the Pythagorean Theorem)

WHAT TO EXPECT ON THE CALCULUS BC FREE RESPONSE QUESTIONS FOR MAY 7TH, 2014

6	<p>Analyze a Table of Values – Increasingly Common</p> <ul style="list-style-type: none"> • It's a good idea to quickly graph the table of values. • By giving a table of values, they can ask: • Reimann Sums (Left-endpoint, Right-endpoint, Midpoint, Trapezoids) • Intermediate Value Theorem (IVT) • Mean Value Theorem (MVT) or Rolle's Theorem • Average (Y) Value Theorem • Rate of Change (slope = derivative) <p>Special Note: Has been on the exam each of the last FIVE years.</p>
7	<p>Equation(s) given as rates of change</p> <ul style="list-style-type: none"> • Typically an equation will be given that is a rate of change. Keep in mind that it is a derivative! • If it's a graphing calculator problem, graph the equation and keep in mind that it is a derivative! • Use calc 1 (value) to calculate rate of change. • Use calc 6 or math 8 (derivative) to calculate if rate of change is increasing (+) or decreasing (-) • Use calc 7 or math 9 (integral) to calculate total amount over time. <ul style="list-style-type: none"> ○ Remember initial amount! • You should notice that this problem is similar to number 8 below. <p>Special note: These problems include: Amusement Park, Sand, Mosquitoes, Traffic Flow, etc.</p>
8	<p>Particle Problem</p> <ul style="list-style-type: none"> • If they don't give a graph of velocity, then expect a particle problem where velocity is given. • If it's a calculator problem (likely), then graph velocity {be careful of endpoints} • Use calc 1 (value) to calculate velocity • Use calc 6 or math 8 (derivative) to calculate acceleration • Use calc 7 or math 9 (integral) to calculate change in position. <ul style="list-style-type: none"> ○ Remember initial position! • Total Distance traveled = $\int_a^b \text{velocity} dt$ • Particle changes direction when $v(t)$ changes signs. • Speed = velocity • Remember: Velocity is increasing when $a(t) > 0$. • Speed is increasing when $v(t)*a(t) > 0$ and Speed is decreasing when $v(t)*a(t) < 0$ <p>Special note: Concept of <u>Absolute Max</u> and <u>Absolute Min</u> could be tested on this problem. Remember to ALWAYS check the endpoints (show table of values)!</p>
9	<p>Slope Fields/Eulers</p> <ul style="list-style-type: none"> • Variable Separable would be included in this one. It's usually worth 5 of the 9 points so make sure that you separate the variables correctly!!!! • EULER'S METHOD IS TYPICALLY ASKED WITH THIS ONE • Euler's Method uses the same equation as the one for linearization \Rightarrow $y(x) \approx y(a) + f'(a)dx$

WHAT TO EXPECT ON THE CALCULUS BC FREE RESPONSE QUESTIONS FOR MAY 7TH, 2014

Other Items to Review:

1. Logistic Growth

- $\frac{dy}{dt} = ky(A - y) \Rightarrow \boxed{y = \frac{A}{1 + Ce^{-Akt}}}$
 - y is the total number of things at time t .
 - $A - y$ is the number of people that don't have "things" at time t .
 - k is the constant of proportionality.
 - A is the carrying capacity (max value of y)
 - $\frac{dy}{dt}$ is a maximum (y growing fastest) when $y = \frac{1}{2}A$
 - $\lim_{t \rightarrow \infty} y(t) = A$
-

2. Implicit Differentiation and/or Variable Separable

- Expect one problem where you will have to do **implicit differentiation** or **variable separable**
 - Remember that a **horizontal** tangent line will have a slope of zero when the numerator of the derivative is 0 and the denominator is not 0.
 - Remember that a **vertical** tangent line will have a slope of undefined when the denominator of the derivative is 0.
-

3. 2nd Derivative test

- Remember that a derivative sign chart is NOT acceptable for justifications.
 - You must state, for example, "Relative Max at $x = 5$ because $f'(x)$ changes from positive to negative at $x = 5$."
 - Be prepared to use the **2nd Derivative test**.
 - For example, "Relative Max at $x = 5$ because $f'(5) = 0$ and $f''(5) < 0$."
-

4. Related Rates

- Make a table of values to keep things organized.
 - 2007 #5 Related Rates problem that incorporated:
Table of Values, Riemann Sums, Linear Approximation, Concept of under/over estimation (Concavity).
 - 2008 Problem #3 (Oil Problem)
-

Remember:

1. Put the work for part A in part A box, etc. Don't cram entire problem into one box!
 2. Make sure to have a watch so you can keep track of your 30 or 60 minutes!
 3. Read over, "Things I Wish I Could Have Told Them" **AGAIN!**
 4. You have been taught **EVERYTHING** that you need in order to earn a **FIVE**. Trust yourself!
 5. Relax. Take a deep breath. You are totally prepared you're going to do **GREAT !!!!!**
-

Final AP Review:

Look at my website link: [Calc2 -- Notes for final review](#).

Look at my website link: [AP Exam Prep](#) for more practice!